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Debt and International Finance

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Debt, Debt Relief, and Growth

A Bargaining Approach

Daniel Cohen
and
Thierry Verdier

Accumulation of reserves and debt-equity swaps can help a debtor country alleviate the distortionary burden of taxing its citizens. But caveats and qualifications apply.

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Debtor countries in the 1980s paid creditors with taxes the governments had to levy on their citizens. Because taxes must be collected in a distortionary way, however, governments are tempted to "up-front" the adjustment effort. Doing so helps relieve investment and growth of the burden of expected future taxation.

Governments have two ways of up-fronting: accumulating reserves and engaging in an equity swap. Cohen and Verdier compare these methods with a constant rescheduling agreement. In the rescheduling agreement, it is assumed that no reserves can be accumulated and that all tax collections go to the creditors. Their findings:

Rescheduling agreement. A "memoryless" (past-independent) rescheduling game was studied. It is Pareto-inefficient. Two Laffer curves can take place. In one, the lenders would want to reduce the vulnerability of the debtor to their sanctions (that is, required taxation). In the other, the debtor would actually prefer less growth than more.

The role of reserves. Use of reserves can improve a country's welfare, over the case of the rescheduling agreement. The country must, however, be able to commit itself to a tax rate before negotiations start. Otherwise, reserves are useless.

Debt-equity swaps. The outcome always Pareto-dominates the outcome of the rescheduling equilibrium. Banks always gain a fraction of the country's capital above the share of output that they gain in the rescheduling equilibrium. Thus banks are relatively less "impatient" than the country to reach a debt-relief agreement.

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Debt, Debt Relief, and Growth: A Bargaining Approach

by
Daniel Cohen
(CEPREMAP, Paris and CEPR, London)
and
Thierry Verdier
(DELTA, Paris)

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I - INTRODUCTION

The net transfers that the government of the indebted LDCs were asked to make to their creditors in the 1980's had to be taxed (one way or another) on their residents. Taxes are bad, because they must be collectea in a distortionary way, and this explains why governments are usually tempted to <u>up-front</u> the adjustment effort that they must undertake so as to relieve (as much as possible) investment and growth from the burden of expected <u>future</u> taxation. (Surprise hyperinflation is often a favorite candidate for such manipulation of the time pattern of taxes).

Upfronting the burden of the adjustment, however, is only good if the debtor country can make sure that it can trade more payments to-day against less in the future. Assume however that the debt which the country owes is infinite (in face value terms). The trade-off between current and later payments can only be achieved through one of the two following channels: 1) the accumulation of reserves by the country which may help it separate the decision to tax the economy from the decision to service the debt of assuming, obviously, that the reserves cannot be seized). 2) a comprehensive debt relief agreement which upfronts the burden of the adjustment (in the paper we analyze the outcome of a massive debt equity swap). It is the comparison of each of these channels to a constant rescheduling agreement which forms the core of the paper.

The analysis in this paper draws on the sequential bargaining approach to negotiation pioneered by Rubinstein (1982) and already applied to the LDC debt problem by Bulow and Rogoff (1989), Fernandez and Rosenthal (1990) and O'Connell (1988) among others. This approach is merged with the (endogeneous) growth model of debt repudiation of Cohen and Sachs (1986) and with the "tax model" of debt of Helpman (1988a and 1988b). Contrarily to the model of debt rescheduling of Bulow and Rogoff, the rescheduling equilibrium that we obtain is typically Pareto-inefficient because of the bad timing of taxation which it induces. Our model will exhibit -in addition to this basic inefficiency- two potential Laffer curves effects. One is the usual disincentive effect of taxation that may be counterproductive to both the lenders and the debtor. Another one is a case of immiserizing growth that is detrimental to the debtor only. To the extent (as we shall prove) that the bargaining power of the debtor is reduced by a faster growth rate, we shall see that it may indeed happen that a good news for growth (say a better productivity of capital) turns out to be a bad news for the debtor's welfare (and a good news for the creditors only).

Assuming away these two Laffer curves effect, we shall proceed to investigate how the Pareto-inefficiency of the bargaining equilibrium could be removed.

We first investigate whether the debtor can raise its welfare by accumulating reserves (assumed now to be non seizable by the creditors) which may help it separate (and redesign optimally) the decision to tax their domestic economy and pay their creditors. We shall see thataccumulating reserves may help the country raise its bargaining stance against the creditors but only if it can commit itself to a tax schedule before the negotiation starts.

We then examine the welfare implications of a comprehensive debt-equity swap. While always Pareto-improving, we show that the deal always turns out to be relatively more to the advantage of the banks than to the advantage of the country. The intuition is simply the following. As long as no deal is struck, the country must invest alone (albeit inefficiently). The banks are therefore relatively less impatient than the country to negotiate a deal.

Section II sets up the model. Section III analyzes the outcome of a memoriless rescheduling agreement. Section IV investigates the role of reserves. Section V concludes with the analysis of a debt equity swap.

II - FRAMEWORK OF ANALYSIS

1 - Let us first describe the country's economy. Following Cohen and Sachs (1986) and Cohen (1991), we assume that the country has access to a technology of production which can be characterized as follows. Production shows a constant return to scale technology:

(1) $Q_t = a K_t$

in which K_t is the stock of installed capital. K_t evolves through a law of motion:

(2) $K_{t+1} = K_t(1-d) + I_t$

in which d is the depreciation rate and I_t is the flow of new installments. It can be obtained through an installment technology where

(3) $J_t = I_t(1 + \frac{1}{2} \phi \frac{I_t}{K_t})$

have to be spent in order to get I_t new capital. Because of the constant return to scale nature of the technology of production with respect to the storable asset, the model exhibits an endogeneous (but non exploding) growth feature qualitatively similar to that analyzed in Romer (1986).

Investment and production is undertaken by a representative private agent. The government's resources are obtained through taxes on GDP. Call θ_t the tax rate at time t, government's resources are given by

(4) $T_t = \theta_t Q_t$

The government's utility is that of the representative agent and is assumed to be a linear function of present and future consumption (we do not analyze, here, the effect of finite intertemporal elasticity of substitution). We let

(5) $U_0 = \sum_0^{\infty} \beta_1^t C_t$

such utility, in which $C_t=Q_t-J_t-T_t$ is the net consumption of the representative private agent at time t.

2 - Let us now describe the international environment. Throughout all this paper we shall assume that the country owes an infinite debt to its creditors (the banks). The creditors and the debtor must negotiate

how much money the debtor will be willing to pay. Let us assume that the banks are only interested in maximizing the present discounted value of the net transfers (P_t) performed by the debtor. Call V_0 this value. If β_0 is the discount factor of the banks :

$$(6) \quad V_0 = \sum_{0}^{\infty} \beta_0^t P_t$$

In section V we assume that the (net) payments by the debtor and the government resource may diverge by allowing the debtor to accumulate (non-seizable) reserves. Here, let us simply assume that all resources collected by the government are paid to the creditors and let the negotiation be directly carried over the tax rate θ_t which the government is required by the debtor to impose on the private agents; let $P_t=\theta_t Q_t$ the net transfers performed by the debtor.

3 - Let us now describe the bargaining structure out of which the payment will be extracted from the country. We shall slightly diverge from Rubinstein alternative offer structure and consider the two following games. Call G_0 the games in which the banks make an offer to the country. If the country accepts to pay whichever transfer P was asked by the bank, we go to the next period and the banks make another offer. If the country refuses the offer from the banks, two things happen. On the one hand, the banks can impose (without a loss to themselves) sanctions to the debtor. On the other hand, the period after, it becomes the country's turn to make an offer. If accepted, we move to the next stage and the country makes another offer; if refused, sanctions are imposed on the country and it becomes the banks' turn to make an offer. Presented in extensive form, the game evolves as follows:

$$
\begin{array}{ccc}
 & G_0 = \text{Banks make offers} & \\
\text{If yes} \swarrow & & \searrow \text{If no. Sanctions } \lambda Q_t \text{ are imposed} \\
G_0 & & G_1
\end{array}
$$

$$
\begin{array}{ccc}
 & G_1 = \text{Country makes offers} & \\
\text{If yes} \swarrow & & \searrow \text{If no. Sanctions } \lambda Q_t \text{ are imposed} \\
G_1 & & G_0
\end{array}
$$

III - CHARACTERISATION OF THE MEMORILESS SUBGAME PERFECT EQUILIBRIUM

1 - In this section, we shall restrict our attention to the case when only memoriless (past independent) strategies can be implemented. Furthermore, we shall keep the assumption that no reserves can be accumulated (say that they could entirely be seized by the creditors) and that all tax collections by the government go to the creditors.

For any equilibrium tax burden θ_t imposed on them, the private agents choose to maximize $\sum_0^\infty \beta_1^t [1-\theta_t-y_t]\ Q_t$ in which y_t is the share of total investment, J_t/Q_t, in GDP. When θ_t is time invariant, one can show that the private agents' strategy is one in which a fixed investment rate and a fixed growth rate is chosen. Call $x = \frac{I}{Q}$ the <u>net</u> investment rate associated to this equilibrium, the growth rate of the economy is then simply:

(7) $n = a\ x\ (\theta) - d$

In which $x(\theta)$ is simply:

$$(8)\ x(\theta) = \underset{x}{\mathrm{Arg\ max}} \frac{1-\theta-x(1+1/2\ \phi\ a\ x)}{1-\beta_1\ [1+a\ x-d]}$$

(In conformity with Romer's model, <u>growth</u> is a positive function of investment).

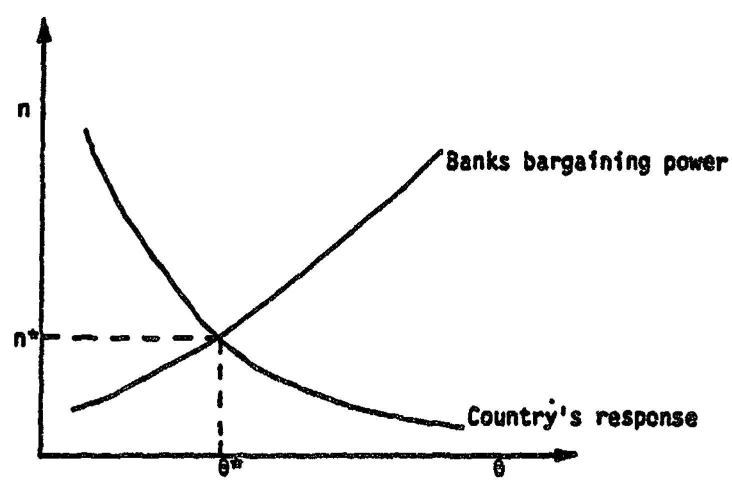
Let us call θ_0 and θ_1 the equilibrium tax rates which are obtained in each of the games G_0 and G_1, (when the banks or the country have a first mover advantage). Let n_0 and n_1 the equilibrium growth rates which they are asociated to, and y_0 and y_1 the corresponding gross investment (J_t/Q_t). Those parameters form a system which must solve the following equations:

$$(9)\ \begin{cases} \text{(a)}\ \dfrac{1-\theta_0-y_0}{1-\beta_1(1+n_0)} = (1-\lambda-y_1) + \beta_1(1+n_1)\dfrac{1-\theta_1-y_1}{1-\beta_1(1+n_1)} \\ \\ \text{(b)}\ \dfrac{\theta_1}{1-\beta_0(1+n_1)} = \beta_0(1+n_0)\dfrac{\theta_0}{1-\beta_0(1+n_0)} \end{cases}$$

Equation (9-a) states that the banks must make an offer θ_0 which is acceptable to the country. (Note that the investment rate, if the country was to refuse the banks' offer, is equal to the investment rate it would choose in the game G_1). Equation (9-b) states that the country must make an offer which is acceptable to the banks. In order to obtain a palatable solution, let us simply assume here that the time between two offers becomes infinitely small. Let $\beta_0 = \frac{1}{1+\delta\ h}$ and let h go towards zero ($\delta_0 \leq \delta_1$). In that case θ_0 and θ_1 converge one towards another. The equilibrium is then simply characterized as follows :

$$
(10)\begin{cases}
\text{(a)}\ \theta = \lambda \dfrac{\delta_1 - n}{\delta_0 + \delta_1 - 2n} \\[2ex]
\text{(b)}\ n = a\ x\ -\ d \\[2ex]
\text{(c)}\ x = \underset{x}{\text{Arg Max}} \dfrac{1 - x(1 + 1/2\ \phi\ a\ x) - \theta}{\delta_1 + d - a\ x}
\end{cases}
$$

One can obtain a simple representation of the equilibrium through the following diagram in growth and payments.



FIGURE

The downward sloping curve (equation (10b) and (10c)) indicates the reaction of the growth rate of the economy to a tax burden θ imposed on the country by the banks. The upward sloping curve (equation 10-a) shows that the bargaining power of the banks is an increasing function of the growth rate of the economy (since $\delta_1 \geq \delta_0$, equation (10-a) shows an increasing relationship between θ and n). Appendix 1 analyzes in detail the conditions under which equations (10) have a unique equilibrium with positive growth. Equation (A1.3) in the appendix offers the relevant condition for existence. We assume it to hold in the sequel.

In the limiting case when $\delta_0=\delta_1$, the bargaining power of the banks is simply such as to impose $\theta = \frac{\lambda}{2}$. The system (10) then becomes a recursive one and growth can be directly calculated out of (10-b) and (10-c) when $\frac{\lambda}{2}$ is substituted to θ. Otherwise, when $\delta_0 < \delta_1$, the share obtained by the banks is always superior to $\lambda/2$.

There are two potential "Laffer Curves" effects associated with the equilibrium depicted in (10). The first one is the usual Laffer-Curve effect. From the banks' view point, all that matter is to maximize $\frac{\Theta}{\delta_0 - n(\Theta)}$. When θ is too large, the disincentive to grow may overcome (from the banks and -a fortiori- from the debtor's view point) the benefit arising from a larger tax rate. In a subgame perfect equilibrium, however, one can <u>not</u> rule out that this Pareto-inefficient outcome will occur. Indeed, in the simple case when $\delta_0 = \delta_1$ the tax rate is $\theta = \frac{\lambda}{2}$. If, say, λ = 100%, the country is totally vulnerable to the sanction imposed by the banks the Laffer Curve effect will be obtained if and only if the optimal tax rate is below 50%. To the extent that <u>current</u> taxation is harmless, the incentive to tax as much as possible on a period-by-period basis explains why a memoriless subgame perfect equilibrium cannot rule out this Laffer Curve effect.

The second Laffer Curve effect which can be obtained in this equilibrium is a case of "immiserizing growth" for the country. As equation (10-a) shows, in effect, the fastest the growth rate, the weaker the bargaining position of the country becomes (whenever $\delta_1>\delta_0$). The intuition behind this result is simply that the larger discount factor of the country makes it more eager to settle for a deal when the benefits of the deal are increased. Now assume that a positive shock, say a permanent reduction of the installation cost of capital (the parameter ϕ in equation (3)) was to occur. The effect can easily be shown to increase the equilibrium growth rate of the economy and the share θ which is obtained by the creditors. The impact on the welfare of the debtor is however ambiguous as these two effects have an opposite effect on the country. The easiest way to see how this "immiserizing growth" effect can be obtained is simply to consider the case when the growth rate of the country is exogeneous (there are no investment decision to be made, x=0). One can show that the formula (10-a) still apply so that the welfare of the debtor is simply:

$$(11)\quad U_{\lambda}(n) = \frac{1-\lambda\,\dfrac{\delta_1-n}{\delta_0+\delta_1-2n}}{\delta_1-n}$$

Take the extreme case when $\lambda=1$, which corresponds to an extremely vulnerable country. In that case, the utility of the country is simply

$$U_1(n) = \frac{\delta_0-n}{[\delta_0+\delta_1-2n]\;[\delta_1-n]}$$

When δ_1 is large enough, this function is a decreasing function of n, and the utility of the country is at its minimum when the growth rate is the fastest, δ_0=n.

Whether these Laffer Curve effect are active or not, however, is not key to our analysis. In all instances, the equilibrium characterized by the system (10) is Pareto-inefficient. Indeed, a constant tax structure can always be improved upon. Because of the distortionary nature of the taxation of output, there is always room for a Pareto-improvement which would allow to trade more payments to the creditors to-day, against less so in the future. The next section will explore what are the consequences of attempting to overcome this inefficiency.

We can summarize the results in this section as follows:

Proposition 1 :There exists a unique time-invariant memoriless subgame perfect equilibrium to the bargaining game between the creditors and the debtor. It is Pareto-inefficient. Two "Laffer-Curve" effects can take place, one where the lenders would want to reduce the vulnerability of the debtor to their sanctions, the other one where the debtor would prefer less growth to more.

III - THE ROLE OF RESERVES

Let us now investigate whether the accumulation of reserves can help the country alleviate the distortionary burden of taxation. In order to analyze this question, we shall make the following alteration to the rules that we played in the rescheduling game of the preceding section. Rather than assuming that the game G_0 or G_1 is indefinitely repeated we shall simply assume that, once an agreement is reached (in the relevant game), the negociation stops and the country routinely transfers at each the payment $\theta_0 Q_t$ or $\theta_1 Q_t$ that were agreed upon. Call $\hat{G}_i$, i=1,2, these new games. In the case when there are no Laffer curves, it is straightforward to see that the equilibrium that is obtained (without reserves) in the games $\hat{G}_i$ is exactly the same as in the game G_i. Indeed, when they make an offer θ_0, the banks must make sure that the country will not prefer refusing their offer and move to the game $\hat{G}_1$. If there are no Laffer curve, they will consequently raise θ_0 up to the maximum value of θ_0 for which (9-a) is satisfied. The same will hold in the game $\hat{G}_1$ so that (9-b) will also be satisfied.

Let us now assume, in addition, that the country can hold reserves that cannot be seized by the creditors . There now exists a potential wedge between the country's current tax burden and the country's payments to its creditors. Call F_t the net flows of accumulated reserves at time t and let τ_t the tax burden imposed on the country. θ_t keeps representing the net transfers to the creditors. We therefore have

$$(17)\ \tau_t Q_t = \theta_t Q_t + F_t$$

We assume in the sequel that reserves pay a rate of interest which is equal to δ_0, the creditors' rate of time preference. In order to simplify the analysis, we shall limit our study to the following two simple games.

1) Game A : The country and the creditor first negociate the value θ_0 or θ_1 that the country is required to pay (stage 1 of Game A). Once

this is done, the country can choose (and credibly commit itself to) a flat tax rate $\tau \leq \theta$ that may differ from the value θ that is agreed upon. We assume that the discrepancy can be financed by a one-shot accumulation of reserves which -perhaps- involves a negative initial negative consumption.

2) Game B : The order of play is reversed with respect to the Game A. First, we assume that the country can commit itself to a flat tax rate τ. Then, the negociation with the creditors start as to how much payments $\theta\, Q_t$ the country should make.

Analysis of the Game A

Let us first analyze the second stage of the game. Assume that the country is (already) required to pay a fraction $\hat{\theta}\, Q_t$ of its output each period. Will it then find it profitable to accumulate reserve so as to alleviate the distortionary burden of taxes ?

Assume that the country builds up initially a stock of reserve R which it uses to reduce (say uniformly) the tax burden on the private economy. The amount of tax reduction that reserves can buy is simply given as :

$$R = \frac{\hat{\theta} - \tau}{\delta_0 - n_\tau}$$

(when writing the equations in continuous time) in which n_τ is the rate of growth which is associated with the tax rate τ. The utility that the country can get out of this accumulation of reserves can be written as

$$U_R = - R + \underset{n}{\text{Max}} \frac{1 - \tau - y(n)}{\delta_1 - n}$$

$$= - \frac{\hat{\theta} - \tau}{\delta_0 - n_\tau} + \frac{1 - \delta - y(n_\tau)}{\delta_1 - n_\tau}$$

Since $\delta_0 \leq \delta_1$, $\frac{\hat{\theta} - \tau}{\delta_0 - n_\tau} \geq \frac{\hat{\theta} - \tau}{\delta_1 - n_\tau}$ so that one can write :

$$(18)\ U_R \leq \frac{1 - \hat{\theta} - y(n_\tau)}{\delta_1 - n_\tau}$$

Now, the utility that the country would obtain by not accumulating reserves is simply :

$$(19)\ U_0 = \underset{n}{\text{Max}}\ \frac{1 - \hat{\theta} - y(n)}{\delta_1 - n}$$

Inspection of equations (18) and (19) shows that $U_0 \geq U_C$. The country cannot raise its welfare by increasing reserves and reducing taxes. The reason is that the potential benefits of lower taxation, a higher growth rate, are partially captured by the lenders.

If one moves back to stage 1 of game A, it is consequently obvious that nothing is changed with respect to the rescheduling game that prevailed without reserves.

Analysis of Game B

Let us now assume that the country is already committed to a pattern of taxation τ when it enters into the rescheduling game $\hat{G}_0$ or $\hat{G}_1$ with the banks.

Call $R(\theta,\tau)$ the amount of reserves that the country must accumulate when a deal θ is struck with the creditors, while it is already committed to imposing a tax rate τ on the economy. Assume that there is a small time interval h between the offers that are made in the games $\hat{G}_0$ and $\hat{G}_1$. One has :

$$R(\theta,\tau) = \frac{\theta - \tau}{\delta_0 - n_\tau}(1 + \delta_0 h)$$

(in which n_τ is the equilibrium growth rate that is achieved by the country when the tax rate τ is implemented).

In the game $\hat{G}_0$ (resp. $\hat{G}_1$) the banks (resp.the country) can make an offer θ_0 (resp. θ_1) that is acceptable if :

$$\left[\frac{1-\tau-y(n_\tau)}{\delta_1 - n_\tau} (1+\delta_1 h) - R(\theta_0,\tau) \right] = [1-\lambda-y(\tilde{n}_0)]h +$$

$$(1+\tilde{n}_0 h) \left[\frac{1-\tau-y(n_\tau)}{\delta_1 - n_\tau} - \frac{1}{1+\delta_1 h} R(\theta_1,\tau) \right]$$

$$\frac{\theta_1}{\delta_0 - n_\tau} (1+\delta_0 h) = \frac{\theta_0}{\delta_0 - n_\tau} (1+\tilde{n}_1 h)$$

where $\tilde{n}_i$ (i=0,1) are the optimum growth rates that are selected by the country in the game G_i when no deal is struck in that game and when it expects to strike a deal at the next round.

Making use of the following equalities :

$$R(\theta_0,\tau) = \frac{\theta_0 - \tau}{\delta_0 - n_\tau} (1+\delta_0 h) \text{ and } R(\theta_1,\tau) = \frac{\theta_1 - \tau}{\delta_0 - n_\tau} (1+\delta_0 h)$$

and letting h go towards zero, one finds that the equilibrium values $(\theta^*, \tilde{n}^*)$ are a solution to :

$$\begin{cases} \dfrac{\theta^*}{\delta_0 - n_\tau} = \dfrac{\delta_1 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \left[\dfrac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \dfrac{1 - \lambda - y(\tilde{n}^*)}{\delta_1 - \tilde{n}^*} \right] + \\[2ex] \qquad \dfrac{\delta_1 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \; \dfrac{\tau}{\delta_0 - \tilde{n}^*} \\[4ex] 1 + \phi\,(\tilde{n}^* + d) = \dfrac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \dfrac{\theta^* - \tau}{\delta_0 - n_\tau} \end{cases}$$

This system yields a solution $\theta^*(\tau)$ and $\tilde{n}^*(\tau)$. Moreover we should only consider domestic tax rates that satisfies $\tau \le \theta^*(\tau)$ in order to have positive reserves.

We can show (in appendix 2) that :

$\forall\, \tau$ such that $0 \le \tau \le \theta^*(\tau)$:

$$U(\tau) = \frac{1-\tau-y(n_\tau)}{\delta_1 - n_\tau} - \frac{\theta^*(\tau)-\tau}{\delta_0 - n_\tau} > U^R$$

where U^R is the utility of the country in the memoriless rescheduling equilibrium of section III and $U(\tau)$ is the level of welfare that the country can get if it expects to be asked to pay $\theta^*(\delta)$ by the bankers. We therefore see that in the game B, the country necessarily obtains a higher utility level than in the game of section III. Hence when the country has the ability to accumulate reserves <u>and</u> to commit itself to a pattern of tax, then it can improve its welfare compared to the case of the rescheduling game. We can summarize our results as follows :

<u>Proposition 2</u> : When it can commit itself, ahead of the negotiation, to a given tax rate, the country can raise its welfare above the memoriless rescheduling equilibrium. Otherwise, reserves serve no purpose.

IV - DEBT RELIEF AND DEBT EQUITY SWAPS

In this section, we now assume that the banks and the country can bargain on a comprehensive debt relief agreement of the following form. We assume that the country can surrender its sovereignty on a piece H_0. If its stock of capital and transfer its property to the creditors. In exchange, the creditors can write-off the debt that is due by the country.

The structure of the bargaining process is the same as the one that we assumed in section II. In the game G_0, the banks offer to write-off the country's debt against a fraction of the country's capital. If the country refuses, it has a pay a cost λQ_0 and may make an alternative offer in the game G_1. If the creditors then refuse the country's offer, sanctions are again imposed on the country and the game switch back to G_0. Whenever a deal is accepted, the game is terminated: the debt is written-off; the creditors and the country each undertake the investment that maximize the welfare that they can obtain from the piece of capital that they eventually own.

Call H_0 and H_1 the offers that are respectively made by the creditors (in Game G_0) and by the debtor (in Game G_1). Assume that the time horizon between two offers is h. One may then write the condition of acceptability of the offers H_0 and H_1 as follows :

$$\text{(22-a)}\quad (1+\delta_1\ h)\left[\frac{1-\tilde{y}}{\delta_1-\tilde{n}}\right]\left[1-H_0\right] = h(1-\lambda-y_b^0) + (1+n_b^0\ h)\left[\frac{1-\tilde{y}}{\delta_1-\tilde{n}}\right](1-H_1)$$

$$\text{(22-b)}\quad H_1\frac{1-y^*}{\delta_1-n^*}(1+\delta_0 . h) = H_0\ \frac{1-y^*}{\delta_0-n^*}(1+n_b^1)$$

in which $\tilde{n}$ and $\tilde{y}$ are respectively the growth rate and the investment rate that the country chooses in order to maximize its utility when the debt is cancelled. Because of the homogeneity of degree 1 of utility with respect to the initial stock of capital, this decision obviously does not depend on the outcome H_0 or H_1 of the game. We can then write $(\tilde{y}, \tilde{n})$ as a solution to :

$$(23)\quad \begin{cases} \tilde{x} = \underset{x}{\text{Arg max}} \dfrac{1-x(1+\frac{1}{2}\,\phi a\;x)}{\delta_1+d-a\;x} \\ \\ \tilde{y} = \tilde{x}(1+\frac{1}{2}\,\phi\,\tilde{x}) \\ \\ \tilde{n} = a\,\tilde{x} - d \end{cases}$$

Similarly, we define n^* and x^* as the corresponding choices of the creditors, when they maximize the utility that they can generate out of H_iK (i=1,2) :

$$(24)\quad \begin{cases} x^* = \underset{x}{\text{Arg max}} \dfrac{1-x(1+\frac{1}{2}\,\phi a\;x)}{\delta_0+d-a\;x} \\ \\ y^* = x^*(1+\frac{1}{2}\,\phi\,x^*) \\ \\ n^* = a\,x^* - d \end{cases}$$

Finally n_b^i and y_b^i (i=0,1) are the optimum growth and investment rates that are selected by the country in the games G_i when no deal is struck in that game and when it expects to strike a deal at the next round.

Substituting H_1 for its value as a function of H_0 from equation (22-b) into equation (22-a) and letting the time interval h shrink to zero, one finds that the equilibrium value H^* is simply :

$$(25)\quad H^* = \frac{\delta_1 - n_b}{\delta_0 + \delta_1 - 2n_b} - \frac{1-\lambda-y_b}{\delta_0 + \delta_1 - 2n_b} \cdot \frac{\delta_1 - \tilde{n}}{1 - \tilde{y}}$$

We can prove the following :

<u>Proposition 3</u> : The outcome of the debt-equity swap always Pareto-dominates the outcome of the rescheduling equilibrium. The banks always gain a fraction of the country's capital that is above the share (of output) that they gain in the rescheduling equilibrium.

The fact the debt-equity swap Pareto-dominates the rescheduling equilibrium is not much of a surprise : by writing down the debt, the banks avoid imposing on the country an inefficient taxation of output. What is more of interest is the comparison of the debt-equity swap outcome with the rescheduling equilibrium. When $\delta_0=\delta_1$, one can see directly from equation (25) that $H^* > \frac{\lambda}{2}$: the banks gain a larger fraction than in the rescheduling game. This hierarchy is shown (in appendix 2) to be always valid. This result may be interpreted as showing that the banks are relatively less "impatient" than the country to reach a debt relief agreement. The intuition behind this result is simply the following. If the country refuses the banks' offer in game G_0 (or if the banks refuse its offer in the game G_1), then the country has to undertake an investment y_b that will be partially ripped-off by the banks in the next round, if an offer is accepted. (The country cannot discriminate between the investment that will raise its own post-deal capital from that which will go to the banks). This explains why the banks are "relatively" less impatient to reach a deal than is the country: as long as no deal is reached, the cost of investment is (albeit inefficiently) carried by the country alone.

APPENDIX 1 : EXISTENCE AND UNIQUENESS OF THE STATIONARY SUBGAME PERFECT EQUILIBRIUM

The two equations caracterizing the equilibrium in equations (10) are :

$$\begin{cases} 1 + \phi\, ax = a\, \dfrac{1-\theta-x\left(1+\dfrac{\phi\, ax}{2}\right)}{\delta_1 + d - ax} & \text{(A1.1)} \\[2ex] \theta = \dfrac{\delta_1+d-ax}{\delta_0+\delta_1+2d-2ax} & \text{(A1.2)} \end{cases}$$

Equation (A1.1) defines a curve $x(\theta)$ (or $n(\theta)$) and equation (A1.2) defines a curve $\theta(x)$ (or $\theta(n)$). It is easy to characterize the curve $x(\theta)$. It is given by :

$$(1+\phi\, ax)(\delta_1 + d - ax) + ax\,\left(1 + \phi\,\frac{ax}{2}\right) = a\,(1-\theta)$$

that is :

$$-\phi\,\frac{a^2 x^2}{2} + ax\,\phi(\delta_1+d) - [a(1-\theta) - (\delta_1+d)] = 0.$$

The relevant solution must satisfy : $0 \le x \le \dfrac{\delta_0+d}{a} < \dfrac{\delta_1+d}{a}$ (in order to have a finite wealth for the country). Hence :

$$x = \frac{1}{a}\left[(\delta_1+d) - (\delta_1+d)\sqrt{1 - \frac{2[a(1-\theta) - (\delta_1+d)]}{\phi\,(\delta_1+d)^2}}\right]$$

and the following conditions on the parameters must hold :

(A1.3) $2\,[a - (\delta_1+d)] < \phi\left[(\delta_1 + d)^2 - (\delta_1 - r)^2\right]$

(in order to have $x \geq \frac{\delta_0+d}{a}$ and $\frac{2[a(1-\theta) - (\delta_1+d)]}{\phi\ (\delta_1+d)^2} < 1$)

It is easy also to see that there exists θ_A and θ_B such that

$x(\theta_A) = \frac{d}{a}$ and $x\ (\theta_B) = 0$

One can write :

$$\theta_A = 1 - \left[\frac{\delta_1+d}{a}\right] - \frac{\phi}{2a}\left[(\delta_1+d)^2 - \delta_1^2\right] \text{ and } \theta_B = 1 - \frac{\delta_1+d}{a}$$

. Hence for $\theta \in (0,\theta_A)$, the growth rate in the economy will be positive

. and for $\theta \in (\theta_A,1)$ the growth rate in the economy will be negative and aways higher than -d. (For $\theta \in (\theta_B,1)$, it is equal to -d).

Moreover : $n\ (0) = ax(0) - d < r$

and $\frac{dn}{d\theta} = \frac{a\ dx}{d\theta} = \frac{-\ a}{(\delta+d\ -\ ax)\ \phi} < 0$

Now equation (A1.2) defines a curve $\theta(n)$ which is increasing in n for $n \in \left[0, \frac{\delta + r}{2}\right[$. Indeed, $\frac{d\theta}{dn} = \frac{\delta_1 - \delta_0}{[\delta_1+\delta_0-r_n]^2}\lambda > 0\ \forall n \in \left[0, \frac{\delta_0+\delta_1}{2}\right[$.

Moreover we have that $\forall n \in [-d,n(0)] : \theta(n) \in [0,\lambda]$ and $\forall\ \theta \in [0,\lambda]$, $n(\theta) \in [-d,n(0)]$

hence : the function $\varphi : [-d,n(0)] \rightarrow \mathbb{R}$

$$n \longrightarrow n[\theta(n)]$$

is a continuous function from $[-d,n(0)]$ into $[-d,n(0)]$.There is consequently a fixed point $\hat{n}$ such that $\varphi(\hat{n}) = \hat{n}$. Moreover since $\varphi'(n) = \frac{dn}{d\theta}\frac{d\theta}{dn} < 0$ and φ is strictly decreasing over $[-d,n(0)]$ one sees that the fixed point is unique. We have therefore shown that and the subgame stationary equilibrium exists and is unique and a solution to :

$$. \ \hat{\theta} = \theta(\hat{n})$$

$$. \ \hat{n} = \text{Argmax} \frac{1-\hat{\theta}-y}{\delta_1 - n}$$

QED

Notes :

. if $\theta_A < \frac{\delta_1}{\delta_0+\delta_1} \lambda$ then $\hat{n}$ is negative with $-d \leq \hat{n} \leq 0$ and

and $\hat{\theta} \in \left[\frac{\lambda}{2} ; \frac{\delta_1}{\delta_0+\delta_0} \lambda \right]$

. if $\theta_A \geq \frac{\delta_1}{\delta_0+\delta_1}$ λ then $\hat{n}$ is positive and $\hat{\theta} \in \left[\frac{\delta_1}{\delta_1+\delta_0} \lambda ; \lambda \right[$

APPENDIX 2 - PROOF OF PROPOSITION 2

The acceptability conditions can be written as follows :

$$\frac{\theta^*}{\delta_0 - n_\tau} = \frac{\delta_1 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \left[\frac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \frac{1 - \lambda - y(\tilde{n}^*)}{\delta_1 - \tilde{n}^*} \right] +$$

$$\frac{\delta_1 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \; \frac{\tau}{\delta_0 - \tilde{n}^*} \tag{A2.1}$$

$$1 + \phi\,(\tilde{n}^* + d) = \frac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \frac{\theta^* - \tau}{\delta_0 - n_\tau} \tag{A2.2}$$

From (A2.2), we conclude that for $\tau \leq \theta^*(\tau)$, one has that :

$$\tilde{n}^* \leq n_\tau$$

Moreover : $U(\tau) = \dfrac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \dfrac{\theta^* - \tau}{\delta_0 - n_\tau}$ and :

$$\frac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} - \frac{\theta^* - \tau}{\delta_0 - n_\tau} = \frac{\delta_0 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \left[\frac{1 - \tau - y(n_\tau)}{\delta_1 - n_\tau} + \frac{\tau}{\delta_1 - n_\tau} \right]$$

$$+ \frac{\delta_1 - \tilde{n}^*}{\delta_0 + \delta_1 - 2\,\tilde{n}^*} \; \frac{1 - \lambda - y(\tilde{n}^*)}{\delta_1 - \tilde{n}^*}$$

Thus, since $\delta_0 \le \delta_1$ and $\tau \ge 0$, we have :

$$U(\tau) \ge \frac{\delta_0-\tilde{n}^*}{\delta_1+\delta_1-2\ \tilde{n}^*}\left[\frac{1-y(n_\tau)}{\delta_1 - n_\tau}\right] + \frac{\delta_1-\tilde{n}^*}{\delta_0+\delta_1-2\ \tilde{n}^*}\left[\frac{1-\lambda-y(\tilde{n}^*)}{\delta_1-\tilde{n}^*}\right]$$

$$\ge \frac{\delta_0-\tilde{n}^*}{\delta_1+\delta_0-2\ \tilde{n}^*}\left[\frac{1-y(n_\tau)}{\delta_1 - n_\tau}\right] + \frac{1-\lambda\,\dfrac{\delta_1-\tilde{n}^*}{\delta_0+\delta_1-2\ \tilde{n}^*}-y(\tilde{n}^*)}{\delta_1-\tilde{n}^*}$$

$$+\left[\frac{\delta_1-\tilde{n}^*}{\delta_0+\delta_1-2\ \tilde{n}^*}-1\right]\frac{1-y(\tilde{n}^*)}{\delta_1-\tilde{n}^*}\ .$$

$$U(\tau) \ge \frac{\delta_0-\tilde{n}^*}{\delta_1+\delta_1-2\ \tilde{n}^*}\left[\frac{1-y(n_\tau)}{\delta_1 - n_\tau} - \frac{1-y(\tilde{n}^*_\tau)}{\delta_1-\tilde{n}^*_\tau}\right] +$$

$$\frac{1-\lambda\,\dfrac{\delta_1-\tilde{n}^*}{\delta_0+\delta_1-2\ \tilde{n}^*}-y(\tilde{n}^*)}{\delta_1-\tilde{n}^*}$$

Thus, since $\tilde{n}_\tau \le n^*$, we also have that :

$$U(\tau) \ge \frac{1-\lambda\,\dfrac{\delta_1-\tilde{n}^*}{\delta_0+\delta_1-2\ \tilde{n}^*}-y(\tilde{n}^*)}{\delta_1-\tilde{n}^*}$$

Using (A2.2) and the fact that the equilibrium growth rate n^R obtained in section III is such that :

$$1 + \phi(n^R + d) = \frac{1 - \lambda \dfrac{\delta_1 - n^R}{\delta_0 + \delta_1 - 2\, n^R} - y(n^R)}{\delta_1 - n^R} = U^R ,$$

we can conclude that :

$\tilde{n}^* \geq n^R$

and also that :

$U(\tau) = 1 + \phi(\tilde{n}^* + d) \geq 1 + \phi(n^R + d) = U^R$

Thus $U(\tau) \geq U^R$.   Q.E.D

APPENDIX 3 - POOF OF PROPOSITION 3

Let us prove that the debt-equity swap Pareto-dominates the rescheduling agreement.

Let us first analyze the country's welfare. From the definition of H^* in equation ( ), one can write the country's welfare as :

$$\text{(A2.1)} \quad U^* = (1-H^*)\frac{1-\tilde{y}}{\delta_1 - \tilde{n}} = \frac{1-\tilde{y}}{\delta_1 - \tilde{n}}\,[1-\theta(n_b)] + \frac{1-\lambda_b - y}{\delta_1 - n_b}\,\theta(n_b)$$

in which

$$\text{(A2.2)} \quad 1-\theta(n) = \frac{\delta_0 - n}{\delta_0 + \delta_1 - 2n} \quad ; \quad \theta(n) = \frac{\delta_1 - n}{\delta_0 + \delta_1 - 2n}$$

and n_b is the solution to :

$$\text{(A2.3)} \quad \begin{cases} n_b = ax_b - d \quad ; \quad y_b = x_b(1 + \frac{1}{2}\phi a\, x_b) \quad ; \quad \pi_b = \pi(n_b) \\ 1 + \phi\, x_b = \dfrac{1-\tilde{y}}{\delta_1 - \tilde{n}}\,(1-\theta_b) + \dfrac{1 - y_b - \lambda\,\theta_b}{\delta_1 - n_b} \end{cases}$$

In the particular case when $\delta_0 = \delta_1$, one sees that $\theta = 1/2$.

From (A2.3) one sees first that : $x_\lambda \leq x_b \leq \tilde{x}$ ($x_b = x_\lambda$ is obtained when $\pi_b = 0$; $x_b = \tilde{x}$ is obtained when $\pi_b = 1$). One can also write

$$\text{(A2.4)} \quad 1 + \phi\, x_b = (1-\theta_b)\left[\frac{1-\tilde{y}}{\delta_1 - \tilde{n}} - \frac{1-y_b}{\delta_1 - n_b}\right] + \frac{1 - y_b - \lambda\,\theta_b}{\delta_1 - n_b}$$

To the extent that the first term in bracket in the right-hand side is necessarily positives and since the solution to the equation

$$\text{(A2.5)}\quad 1 + \phi\, x = \frac{1 - y - \lambda\, \theta(n)}{\delta - n}$$

is nothing else but the solution to the rescheduling equilibrium, one sees that $x_b > \hat{x}$ ($\hat{x}$ the rescheduling solution) <u>and</u> that the level of welfare reached by the country (which is nothing else but the right-hand side in equation (A2.4)) is above the rescheduling level (which is the right-hand side of equation A2.5).

Let us now turn to the banks' pay-off.

One can write

$$H^* = \theta_b \left[1 - \frac{1 - y_b}{\delta_1 - n_b} \cdot \frac{\delta_1 - \tilde{n}}{1 - \tilde{y}} \right] + \theta_b \left[\frac{\delta_1 - \tilde{n}}{1 - \tilde{y}} \right] \frac{1}{\delta_1 - n_b} \cdot \lambda$$

Since the first term in bracket is positive (and because $\lambda \leq 1$) one can write

$$H^* \geq \lambda\, \theta_b \left\{ 1 + \frac{\delta_1 - \tilde{n}}{1 - \tilde{y}} \cdot \frac{y_b}{\delta_1 - n_b} \right\}$$

So that

$$H^* \geq \lambda\, \theta_b$$

Since $n_b \geq n_0$, this shows that $H^* \geq \lambda\, \theta_0$: the banks gain a larger fraction of the country's capital stock than the share $\hat{\theta}$ that they obtained in the recheduling game. This proves the second part of proposition 3.

In order to show that this does improve their pay-off one has to see that

$$\frac{\delta_1 - \bar{n}}{\delta_1 - n_b} \geq \frac{\delta_0 - \bar{n}}{\delta_0 - n_b} \text{ (since } \bar{n} > n_b\text{, and } \delta_1 \geq \delta_0\text{)}$$

so that

$$H^* \geq \lambda\, \theta_b \left[1 + \frac{\delta_0 - \bar{n}}{1 - \tilde{y}} \cdot \frac{1}{\delta_0 - n_b} \cdot y_b\right]$$

This shows that the pay-off of the banks which is :

$$\text{(A2.6) } V^* = H^* \frac{1 - y^*}{\delta_0 - n^*} = \left[\lambda\, \theta_b \frac{1 - y^*}{\delta_0 - n^*}\right] + \left[\frac{y_b\, \lambda\theta_b}{\delta_0 - n_b} \left(\frac{\delta_0 - \bar{n}}{1 - \tilde{y}} \cdot \frac{1 - y^*}{\delta_0 - n^*}\right)\right]$$

is certainly above the rescheduling level :

$$V_0 = \frac{\lambda\, \theta_0}{\delta_0 - n_0}$$

Indeed this latter payment can be written :

$$\text{(A2.7) } V_0 = \frac{\lambda\, \theta_0 (1 - y_0)}{\delta_0 - n_0} + \frac{\lambda\, \theta_0 \cdot y_0}{\delta_0 - n_0}$$

and the first and the second term in equation (A2.6) respectively dominate the corresponding terms in equation (A2.7).

Q.E.D

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